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SUMMARY

A method for determining three-dimensional steady-state temperature distributions is presented for plates in which heat is generated and conducted throughout the plate material. The solution for the temperature distribution is given in terms of a power series with respect to the plate thickness coordinate. The coefficients of the power series are dependent upon the temperature distribution over the plate surfaces. Specific attention is given to the determination of temperature distributions and plate deformations resulting from constant heat generation and from the absorption of electromagnetic radiation.

INTRODUCTION

The phenomenon of heat generation in materials occurs as a result of the conversion of energy into heat in such processes as electrical-current conduction, chemical reactions, and absorption of electromagnetic radiation. Heat generation within a structural material causes nonlinear temperature gradients to develop, and may result in undesirable thermal deformations or high thermal stresses. Some recent investigations have been carried out (refs. 1 to 6) to determine transient one-dimensional temperature distributions resulting from the absorption of various forms of electromagnetic radiation. In these analyses special attention was given to the determination of the thermal stresses and deformations in plate and shell configurations resulting from nuclear radiation (refs. 1 to 4). Examination of the equations governing plate and shell deformations, however, indicates that thermal deformation is also dependent upon the temperature gradients in the directions normal to the thickness coordinate. (See refs. 3 and 7 to 10.) It is therefore of interest to determine not only the one-dimensional (i.e., thickness direction) but also the three-dimensional temperature distributions throughout the structural material.

The work presented herein develops a method for determining three-dimensional steady-state temperature distributions caused by internal heat generation in plates. The corresponding expressions for the thermal-deformation parameters are presented for plates and shells. Specific examples are given for heat generation caused either by constant heat generation or by absorption of electromagnetic radiation.

SYMBOLS

$a_n(x,y), b_n(x,y)$ coefficients of power series in z

$A(x,y,z)$ heat generated at any point within the plate per unit time per unit volume

$b_0 = T(x,y,0)$

$b_1 = \frac{\partial T}{\partial z}(x,y,0)$

B_n Bernoulli numbers

C_n, D_m constants

E_n Euler numbers

$f(x,y,z) = -\frac{A(x,y,z)}{k}$

h plate thickness

I intensity of electromagnetic radiation

I_o energy incident on plate surface $z = h/2$

k thermal conductivity of plate material

m,n integers in power series

$P_o = \frac{1}{2}\left(T_{h/2} - T_{-h/2}\right)$

T temperature change from an initial state

$T_{h/2} = T(x,y,h/2)$

$T_{-h/2} = T(x,y,-h/2)$

$T_o = \frac{1}{2}\left(T_{h/2} + T_{-h/2}\right)$

x,y,z rectangular coordinates

β linear absorption coefficient

$\beta_n = \left(2^{2n} - 2\right)B_n$

∇_1^2 differential operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

∇^2 differential operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

∇^m differential operator $(\nabla^2)^{m/2}$; ∇^0 indicates no differential operation intended

∇^n differential operator $(\nabla^2)^{n/2}$; ∇^0 indicates no differential operation intended

φ_0 plate temperature parameter $\int_{-h/2}^{h/2} T\,dz$

φ_1 plate temperature parameter $\int_{-h/2}^{h/2} Tz\,dz$

THEORY

The following analysis presents a general series approach for determining three-dimensional steady-state temperature distribution in plates. The general series solution is then used, in specific applications, for the determination of plate temperature distributions and the corresponding thermal-deformation parameters.

General Solution of the Heat-Conduction Equation

Steady-state temperature distributions within solid bodies in which conduction is the mode of heat transfer and which have isotropic thermal properties are governed mathematically by Poisson's equation (ref. 11):

$$\nabla_1^2 T = -\frac{A(x,y,z)}{k} \tag{1}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

T temperature at any point

A(x,y,z) heat generated at any point within the plate per unit time per unit volume

k thermal conductivity of plate material (assumed constant)

Solutions of Poisson's equation may be found by means of the Green's function technique. (See ref. 12.) However, application of this method of solution to boundary-value problems is difficult since the solution involves a surface and volume integral over the body geometry. Another analytical method of approach to the solution of Poisson's equation may be formulated by expressing both the temperature and the right-hand side of equation (1) in terms of power series in z. This gives

$$T = \sum_{n=0,1,2,3}^{\infty} b_n(x,y)z^n \tag{2}$$

and

$$-\frac{A(x,y,z)}{k} = f(x,y,z) = \sum_{n=0}^{\infty} a_n(x,y)z^n \tag{3}$$

where functions $a_n(x,y)$ are known because $f(x,y,z)$ is assumed to be known. Substituting equations (2) and (3) into equation (1) and equating the coefficients of like powers of z gives the recurrence relation

$$a_n = \nabla^2 b_n + (n + 1)(n + 2)b_{n+2} \tag{4}$$

With the use of this recurrence relation, the coefficients, b_n, of z^n in equation (2) may be expressed in terms of b_0, b_1, and a_n. Equation (2) then becomes

$$T = \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{1}{n!} \nabla^n b_0 z^n + \sum_{n=0,2,4}^{\infty} z^{n+2} \sum_{m=0,2,4}^{n} (-1)^{m/2} \frac{(n - m)!}{(n + 2)!} \nabla^m a_{n-m} + \sum_{n=1,3,5}^{\infty} (-1)^{(n-1)/2} \frac{1}{n!} \nabla^{n-1} b_1 z^n$$

$$+ \sum_{n=1,3,5}^{\infty} z^{n+2} \sum_{m=1,3,5}^{n} (-1)^{(m-1)/2} \frac{(n - m + 1)!}{(n + 2)!} \nabla^{m-1} a_{n-m+1} \tag{5}$$

where b_0 and b_1 are, respectively, the temperature distribution at the plane $z = 0$ and the temperature gradient in the z-direction at the plane $z = 0$. If the functions b_0 and b_1 are known, the temperature distribution within the body is determined by equation (5). The functions b_0 and b_1 are determined from the temperature boundary conditions.

Determination of Coefficients b_0 and b_1 in the General Solution

For the plate shown in figure 1, the temperature distribution within the plate is largely dependent on the temperature boundary conditions on the large surfaces $(z = \pm h/2)$ and is relatively unaffected by the temperature boundary conditions along the edges of the plate. Therefore, the edge boundary conditions will not be considered and the coefficients b_0 and b_1 will be functions of the boundary temperatures $T_{h/2}$ and $T_{-h/2}$ on the large surfaces of the plate and of the internal heating function $a_n(x,y)$.

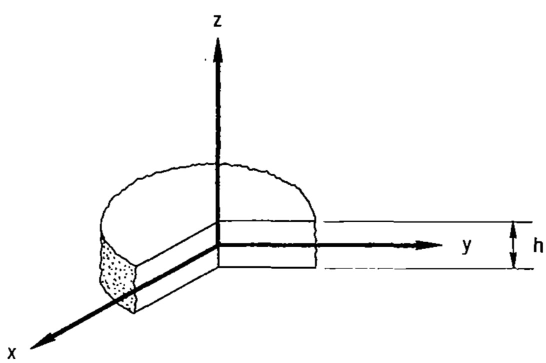


Figure 1.- Coordinate system associated with plate.

In order to determine b_0 and b_1 in terms of $T_{h/2}$, $T_{-h/2}$, and $a_n(x,y)$ it is convenient to define two boundary-temperature functions T_o and P_o in terms of the specified boundary temperatures

$$T_o = \frac{1}{2}\left(T_{h/2} + T_{-h/2}\right) \tag{6}$$

$$P_o = \frac{1}{2}\left(T_{h/2} - T_{-h/2}\right) \tag{7}$$

Expressing T_o and P_o in terms of b_0, b_1, and a_n by the use of equation (5) gives

$$T_o = T_o^* + \sum_{n=0,2,4}^{\infty} \left(\frac{h}{2}\right)^{n+2} \sum_{m=0,2,4}^{n} (-1)^{m/2} \frac{(n - m)!}{(n + 2)!} \nabla^m a_{n-m} \tag{8}$$

$$P_o = P_o^* + \sum_{n=1,3,5}^{\infty} \left(\frac{h}{2}\right)^{n+2} \sum_{m=1,3,5}^{n} (-1)^{(m-1)/2} \frac{(n - m + 1)!}{(n + 2)!} \nabla^{m-1} a_{n-m+1} \tag{9}$$

where

$$T_o^* = \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{1}{n!} \nabla^n b_0 \left(\frac{h}{2}\right)^n \tag{10}$$

and

$$P_o{}^* = \sum_{n=1,3,5}^{\infty} (-1)^{(n-1)/2} \frac{1}{n!} \nabla^{n-1} b_1 \left(\frac{h}{2}\right)^n \tag{11}$$

It should be noted that equations (8) and (9) establish relationships between T_o and b_0 independent of b_1, and between P_o and b_1 independent of b_0. Expressions for b_0 and b_1 may be obtained in terms of $T_o{}^*$ and $P_o{}^*$, respectively, as outlined in appendix A. These expressions are given by

$$b_0 = \sum_{n=0,2,4}^{\infty} \frac{E_n}{n!}\left(\frac{h}{2}\right)^n \nabla^n T_o{}^* \tag{12}$$

$$b_1 = \frac{2P_o{}^*}{h} + \sum_{n=3,5,7}^{\infty} \frac{\beta_{(n-1)/2}}{(n-1)!}\left(\frac{h}{2}\right)^{n-2} \nabla^{n-1} P_o{}^* \tag{13}$$

where the constants E_n are the Euler numbers, and the constants $\beta_{(n-1)/2}$ are related to the Bernoulli numbers (see eq. (A9)).

Thus the functions b_0 and b_1 may be found in terms of T_o, P_o, and $a_n(x,y)$ by first determining $T_o{}^*$ and $P_o{}^*$ from equations (8) and (9), and substituting the resulting expressions into equations (12) and (13). When the solutions for b_0 and b_1 are substituted into equation (5), the resulting temperature distribution will satisfy the governing Poisson equation (eq. (1)) and the specified temperature boundary conditions on the large surfaces of the plate. Appendix B presents general expressions for the temperature distribution in which $T_{h/2}$, $T_{-h/2}$, and the coefficients $a_n(x,y)$ in the series expansion for the heat-generation function are limited to either biharmonic functions or harmonic functions.

Determination of the Plate Thermal-Deformation Parameters

Equations governing the thermal deformations of plates may be found in numerous places in the literature. (See refs. 3 and 7 to 10.) The basic assumptions used in these analyses require the definition of two temperature parameters which appear in the equilibrium equations and boundary conditions governing plate deformation. These parameters are given by

$$\varphi_0 = \int_{-h/2}^{h/2} T \, dz \tag{14}$$

$$\varphi_1 = \int_{-h/2}^{h/2} Tz \, dz \tag{15}$$

and are proportional, respectively, to the average temperature distribution through the plate thickness and the first moment of the temperature distribution with respect to the midplane of the plate. For small-deflection plate theory the parameter φ_0 appears only in the equations for midplane stresses and in-plane deformations. Similarly, the parameter φ_1 appears only in the equations for bending stresses and out-of-plane deformations. (See ref. 10.)

Expressions for the plate thermal-deformation parameters may be obtained in terms of b_0 and b_1 by substituting equation (5) into equations (14) and (15) and performing the indicated integrations. The resulting expressions for φ_0 and φ_1 are given by

$$\varphi_0 = \varphi_0^* + 2 \sum_{n=0,2,4}^{\infty} \frac{\left(\frac{h}{2}\right)^{n+3}}{n + 3} \sum_{m=0,2,4}^{n} (-1)^{m/2} \frac{(n - m)!}{(n + 2)!} \nabla^m a_{n-m} \tag{16}$$

$$\varphi_1 = \varphi_1^* + 2 \sum_{n=1,3,5}^{\infty} \frac{\left(\frac{h}{2}\right)^{n+4}}{n + 4} \sum_{m=1,3,5}^{n} (-1)^{(m-1)/2} \frac{(n - m + 1)!}{(n + 2)!} \nabla^{m-1} a_{n-m+1} \tag{17}$$

where

$$\varphi_0^* = 2 \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{\nabla^n b_0}{(n + 1)!} \left(\frac{h}{2}\right)^{n+1} \tag{18}$$

and

$$\varphi_1^* = 2 \sum_{n=1,3,5}^{\infty} (-1)^{(n-1)/2} \frac{\nabla^{n-1} b_1}{(n + 2)(n!)} \left(\frac{h}{2}\right)^{n+2} \tag{19}$$

The functions φ_0 and φ_1 may thus be determined from equations (16) and (17) after obtaining expressions for b_0 and b_1 which are functions of the temperature boundary conditions and the heating function.

From the expression for the plate thermal-deformation parameter φ_0, it can be shown that φ_0 is dependent only on the boundary temperature function T_0 and the coefficients a_n for even values of n. Similarly, it can be shown that the plate thermal-deformation parameter φ_1 is dependent only on P_0 and the coefficients a_n for odd values of n. Therefore, when the heating function A is represented by an even function with respect to the midplane of the plate, the plate thermal-deformation parameter φ_1 is unaffected by heat generation.

APPLICATIONS

Constant Heat Generation

General solution.- For the case of constant heat generation, the coefficients a_n for the series defined by equation (3) are

$$\left.\begin{aligned} a_0 &= -\frac{A}{k} \\ a_n &= 0 \qquad\qquad (n > 0) \end{aligned}\right\} \tag{20}$$

The temperature distribution given by equation (5) thus reduces to

$$T = \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{\nabla^n b_0 z^n}{n!} + \sum_{n=1,3,5}^{\infty} (-1)^{(n-1)/2} \frac{\nabla^{n-1} b_1 z^n}{n!} - \frac{Az^2}{2k} \tag{21}$$

The relationships between T_o and b_0 and between P_o and b_1 are obtained from equations (8) and (9) and are given by

$$\left.\begin{aligned} T_o &= T_o^* - \frac{h^2 A}{8k} \\ P_o &= P_o^* \end{aligned}\right\} \tag{22}$$

The thermal-deformation parameters are obtained from equations (16) and (17) and are given by

$$\left.\begin{aligned} \varphi_0 &= \varphi_0^* - \frac{h^3 A}{24k} \\ \varphi_1 &= \varphi_1^* \end{aligned}\right\} \tag{23}$$

Equations (23) indicate that φ_0 is the only plate temperature parameter affected by constant heat generation. Thus, for small deformations, since A is an even function, the constant internal heat generation affects only the in-plane plate deformation. In addition, the effect of constant internal heat generation is to change the value of φ_0 over the entire plate by a constant amount.

Harmonic solution.- If, in addition to a constant A, it is assumed that $\nabla^2 T_{h/2}$ and $\nabla^2 T_{-h/2}$ are zero, then a simpler form of plate temperature distribution is obtained from equation (B16) of appendix B as

$$T = \frac{A}{2k}\left(\frac{h^2}{4} - z^2\right) + \frac{2P_o z}{h} + T_o \tag{24}$$

The corresponding plate thermal-deformation parameters are obtained from equations (B17) and (B18) and are given by

$$\varphi_0 = T_o h + \frac{h^3 A}{12k} \tag{25}$$

and

$$\varphi_1 = \frac{P_o h^2}{6} \tag{26}$$

Absorption of Electromagnetic Radiation

General solution.- An assumption commonly made in calculations involving the absorption of electromagnetic radiation is that dI, the incremental amount of energy absorbed in an incremental thickness dz, is proportional to both the intensity of the electromagnetic radiation I and the thickness dz. (See ref. 13.) With the use of this assumption, the heat generated per unit time per unit volume is given by

$$A(x,y,z) = \beta I_o(x,y) e^{-\beta\left(\frac{h}{2} - z\right)} \tag{27}$$

where I_o is the energy incident and normal to the plane surface $z = h/2$ with no losses due to reflection and β is the linear absorption coefficient.

The heating function A as described by equation (27) is zero for either $\beta = 0$, $\beta \rightarrow \infty$, or $I_o = 0$. When equation (27) is substituted into equation (3), the summation of the power series in z with coefficients a_n is defined as

$$\sum_{n=0}^{\infty} a_n z^n = -\frac{\beta I_o(x,y)}{k} e^{-\beta\left(\frac{h}{2} - z\right)} \tag{28}$$

Expanding the right-hand side of equation (28) in a power series in z and equating like powers of z gives the expression for a_n as

$$a_n(x,y) = -\frac{I_o(x,y) e^{-\beta h/2} \beta^{n+1}}{k n!} \tag{29}$$

Substitution of equation (29) into equation (5) gives the temperature equation as

$$T = \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{\nabla^n b_0}{n!} z^n + \sum_{n=1,3,5}^{\infty} (-1)^{(n-1)/2} \frac{\nabla^{n-1} b_1 z^n}{n!}$$

$$- \frac{e^{-\beta h/2}}{k} \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{\nabla^n I_0}{\beta^{n+1}} \left[e^{\beta z} - \sum_{m=0,1,2,3}^{n+1} \frac{(\beta z)^m}{m!} \right] \tag{30}$$

In order to satisfy the specified temperature boundary conditions, the functions b_0 and b_1 in equation (30) must be determined in terms of T_o^* and P_o^* from equations (12) and (13). First the functions T_o^* and P_o^* are found by substituting equation (29) into equations (8) and (9). After some algebraic manipulation, this gives T_o^* and P_o^* as

$$T_o^* = T_o + \frac{1}{2k} \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{\nabla^n I_o}{\beta^{n+1}} \left[e^{-\beta h} + 1 - 2e^{-\beta h/2} \sum_{m=0,2,4}^{n} \frac{1}{m!} \left(\frac{\beta h}{2}\right)^m \right] \tag{31}$$

and

$$P_o^* = P_o + \frac{1}{2k} \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{\nabla^n I_o}{\beta^{n+1}} \left[-e^{-\beta h} + 1 - 2e^{-\beta h/2} \sum_{m=1,3,5}^{n+1} \frac{1}{m!} \left(\frac{\beta h}{2}\right)^m \right] \tag{32}$$

The functions b_0 and b_1 can then be found by substituting equations (31) and (32) into equations (12) and (13).

The plate thermal-deformation parameters may be found by appropriately integrating equation (30) for φ_0 and φ_1 or by substituting equation (29) into equations (16) and (17). Either method gives

$$\varphi_0 = \varphi_0^* - \frac{2e^{-\beta h/2}}{k} \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{\nabla^n I_o}{\beta^{n+2}} \left[\sinh \frac{\beta h}{2} - \sum_{m=0,2,4}^{n} \left(\frac{\beta h}{2}\right)^{m+1} \frac{1}{(m+1)!} \right] \tag{33}$$

$$\varphi_1 = \varphi_1^* - \frac{2e^{-\beta h/2}}{k} \sum_{n=0,2,4}^{\infty} (-1)^{n/2} \frac{\nabla^n I_o}{\beta^{n+3}} \left[\frac{\beta h}{2} \cosh \frac{\beta h}{2} - \sinh \frac{\beta h}{2} \right.$$

$$\left. - \sum_{m=1,3,5}^{n+1} \left(\frac{\beta h}{2}\right)^{m+2} \frac{1}{(m+2)m!} \right] \tag{34}$$

Equations (33) and (34) are the general expressions for φ_0 and φ_1 in terms of b_0 and b_1. (See eqs. (18) and (19).) Appendix B presents a solution which illustrates the general procedure used in obtaining T, φ_0, and φ_1 in terms of T_o, P_o, and I_o.

Solution when $T_{h/2}$, $T_{-h/2}$, and I_o are harmonic.- Two specific cases are presented, one for temperature distributions in which $T_{h/2}$ and $T_{-h/2}$ are prescribed, and one for temperature distributions in which $T_{h/2}$ is prescribed and the surface $z = -h/2$ is insulated.

Temperature distributions in which $T_{h/2}$ and $T_{-h/2}$ are prescribed: Appendix B presents the solutions for T, φ_0, and φ_1 for the case in which $T_{h/2}$, $T_{-h/2}$, and I_o are biharmonic functions. (See eqs. (B19) to (B21).) If $T_{h/2}$, $T_{-h/2}$, and I_o are harmonic functions, the corresponding expressions for T, φ_0, and φ_1 may be obtained from equations (B19) to (B21) by letting

$$\left.\begin{aligned}\nabla^2 T_o &= 0\\ \nabla^2 P_o &= 0\\ \nabla^2 I_o &= 0\end{aligned}\right\} \tag{35}$$

The resulting expressions are

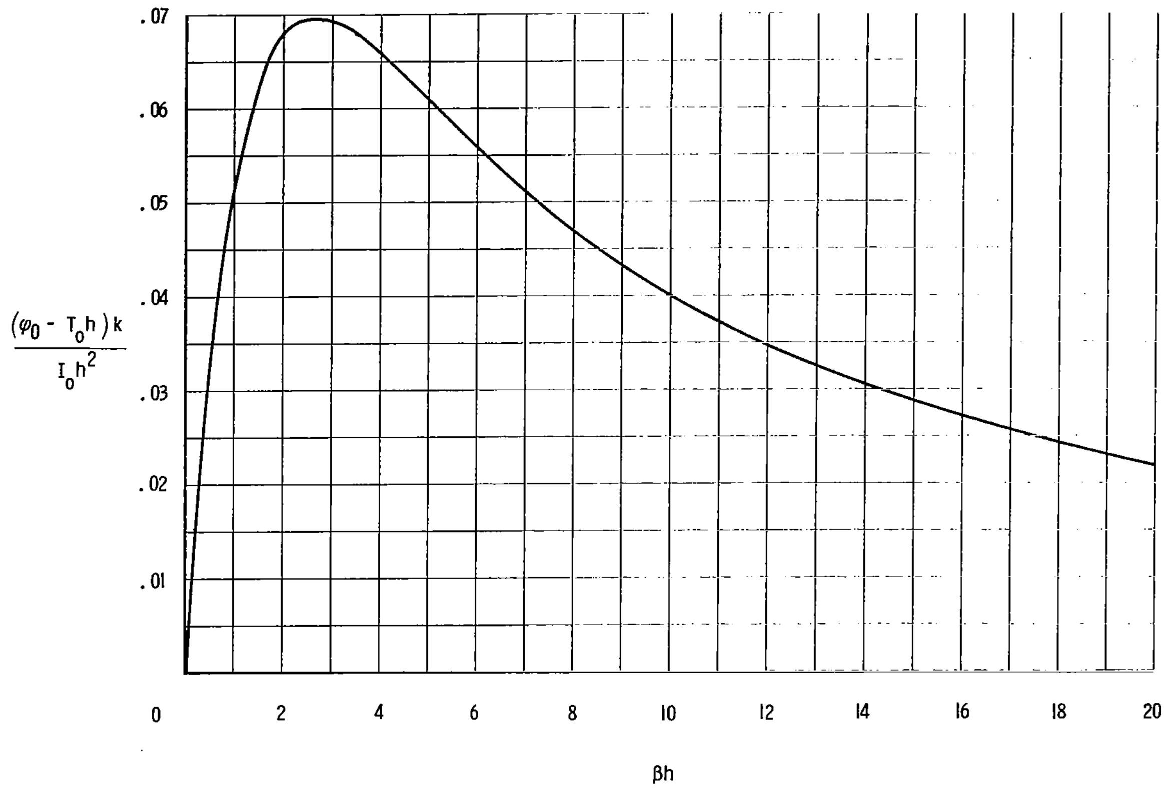
$$T = T_o + \frac{2P_o z}{h} + \frac{I_o}{k\beta h}\left[\left(\frac{h}{2} + z\right) + e^{-\beta h}\left(\frac{h}{2} - z\right) - he^{-\beta\left(\frac{h}{2} - z\right)}\right] \tag{36}$$

$$\varphi_0 = T_o h + \frac{I_o h^2}{2k(\beta h)^2}\left[\beta h - 2 + e^{-\beta h}(\beta h + 2)\right] \tag{37}$$

$$\varphi_1 = \frac{P_o h^2}{6} + \frac{I_o h^3}{12k(\beta h)^3}\left\{(\beta h)^2 - 6\beta h + 12 - e^{-\beta h}\left[(\beta h)^2 + 6\beta h + 12\right]\right\} \tag{38}$$

Equations (37) and (38) indicate that the plate temperature parameters are linearly dependent on the temperature distributions over the large surfaces of the plate and on the magnitude of the incident radiation.

The value of the linear absorption coefficient β also affects the magnitude of the plate thermal-deformation parameters inasmuch as it governs the value of the coefficient of the incident-radiation term. Figures 2(a) and 2(b) show the effect of βh on coefficients of the incident-radiation terms from equations (37) and (38). It may be seen from figure 2(a), where $(\varphi_0 - T_o h)k/I_o h^2$ is plotted against βh, that radiation absorption has the largest effect on φ_0, and hence on the in-plane small-deflection plate deformations, for a value of βh of approximately 2.6. It should be noted that φ_0 is



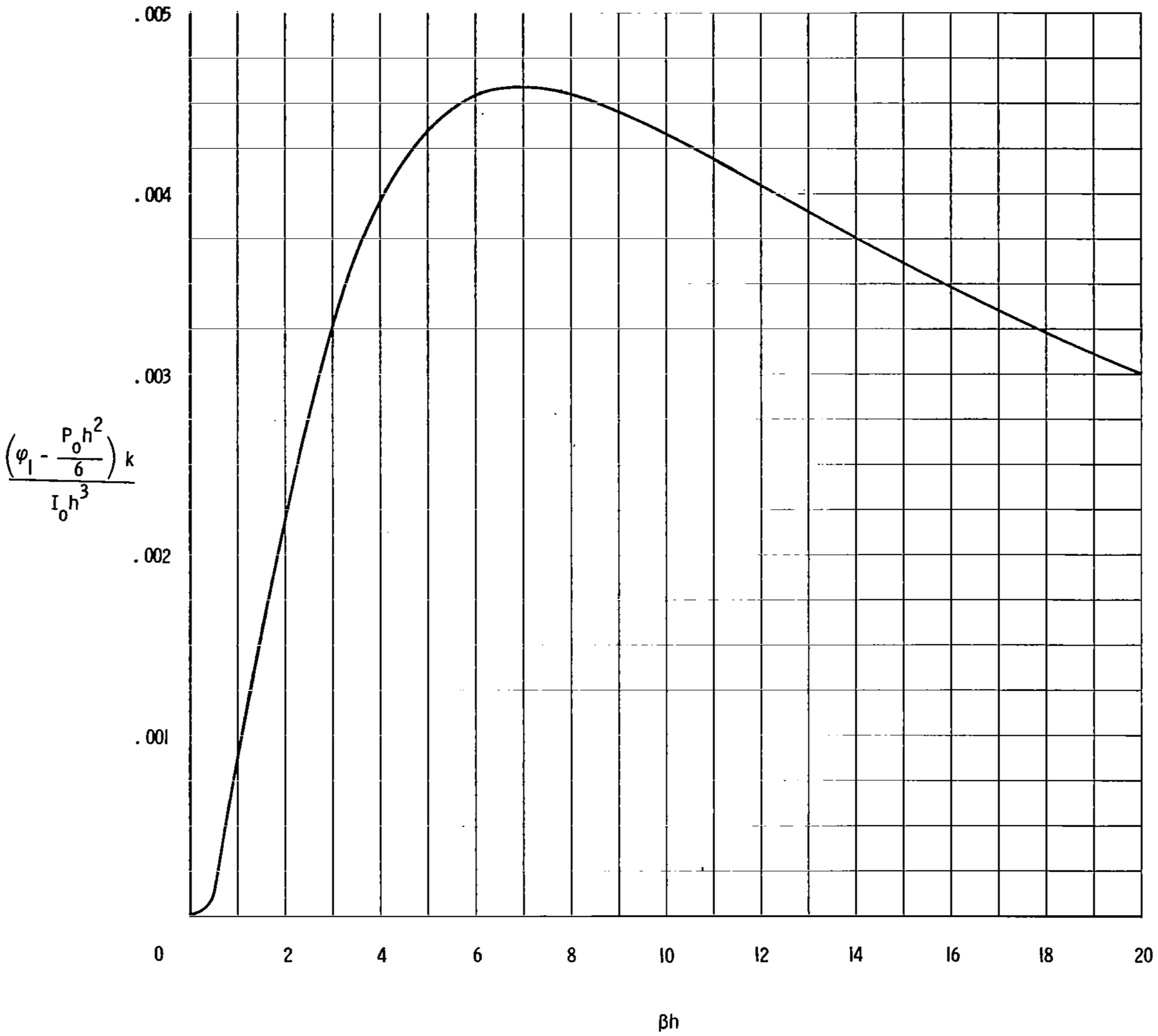
(a) Effect of βh on φ_0.

Figure 2.- Effect of βh on plate temperature parameters for specified boundary temperatures and incident energy distribution.

equal to $T_o h$ when βh is equal to zero. Also, φ_0 approaches $T_o h$ as βh approaches infinity. In these limiting cases no radiation is absorbed within the plate, and the temperature gradient is constant through the thickness of the plate. Figure 2(b) shows the quantity $\left(\varphi_1 - \frac{P_o h^2}{6}\right)k \Big/ I_o h^3$ plotted against βh, and indicates that the quantity φ_1, which determines the out-of-plane small-deflection plate deformations, increases from a value of $P_o h^2/6$ for a value of βh equal to zero until βh is 7.14. Thereafter, φ_1 decreases with increasing βh and approaches $P_o h^2/6$ as βh approaches infinity. As before, the extreme values of βh are cases in which no radiation is absorbed within the plate and the temperature gradient is constant through the thickness of the plate.

Temperature distributions in which $T_{h/2}$ is prescribed and the surface $z = -h/2$ is insulated: For this case the function $T_{-h/2}$ is eliminated from the T_o and P_o terms of equation (36) by using the condition

$$\left.\frac{\partial T}{\partial z}\right|_{z=-h/2} = 0 \tag{39}$$



(b) Effect of βh on φ_1.

Figure 2.- Concluded.

The resulting expression for the temperature distribution is

$$T = T_{h/2} + \frac{I_o}{k}\left[e^{-\beta h}\left(z - \frac{h}{2}\right) - \frac{e^{\beta\left(z - \frac{h}{2}\right)}}{\beta} + \frac{1}{\beta}\right] \tag{40}$$

Because no heat flow is allowed across the surface $z = -h/2$, the temperature gradient through the thickness of the plate must allow heat conduction toward the surface $z = h/2$. Thus the maximum temperature in the thickness direction always occurs at the surface $z = -h/2$. A typical temperature profile, as obtained from equation (40), is shown in figure 3 in terms of the dimensionless temperature parameter $\left(T - T_{h/2}\right)k/I_o h$ and the dimensionless thickness parameter $2z/h$. The magnitude of the temperature difference between the front and back surfaces of the plate is determined by the parameter βh.

Figure 4 shows the quantity $\left(T_{-h/2} - T_{h/2}\right)k/I_o h$ plotted as a function of βh from equation (40). It is seen that a maximum temperature difference between the front and back surfaces will occur at a value of βh equal to 1.8. Thus, appropriate selection of plate material and plate thickness will allow temperatures at the back surface $(z = -h/2)$ to be maximized with respect to the front surface temperature.

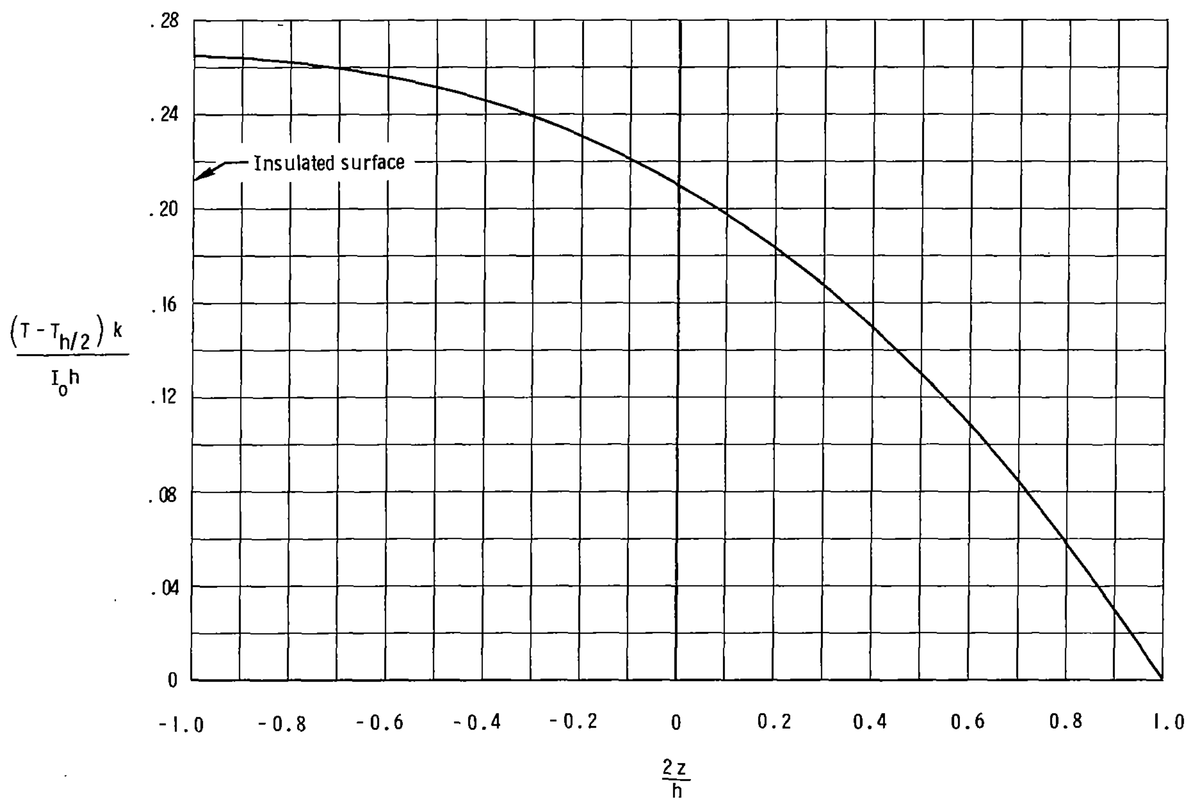


Figure 3.- Temperature profile through plate thickness when $\frac{\partial T}{\partial z} = 0$ at $z = \frac{h}{2}$ and $\beta h = 1.0$.

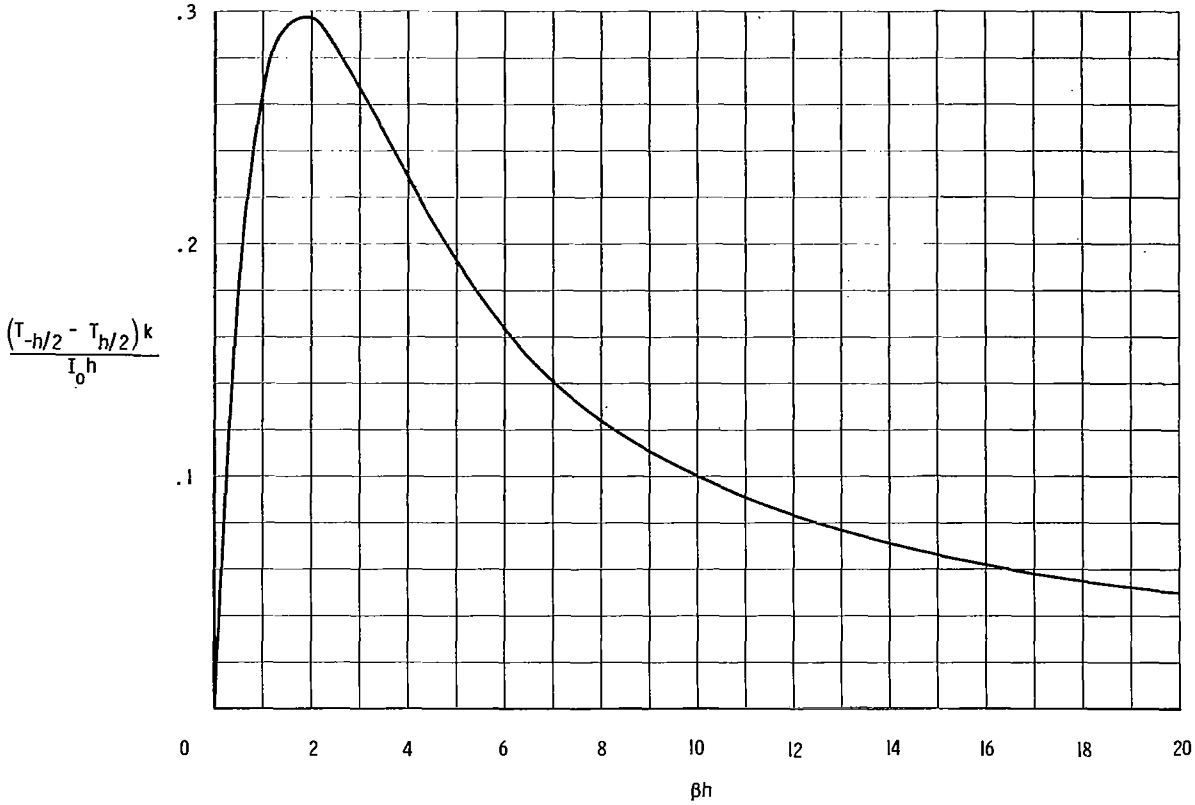


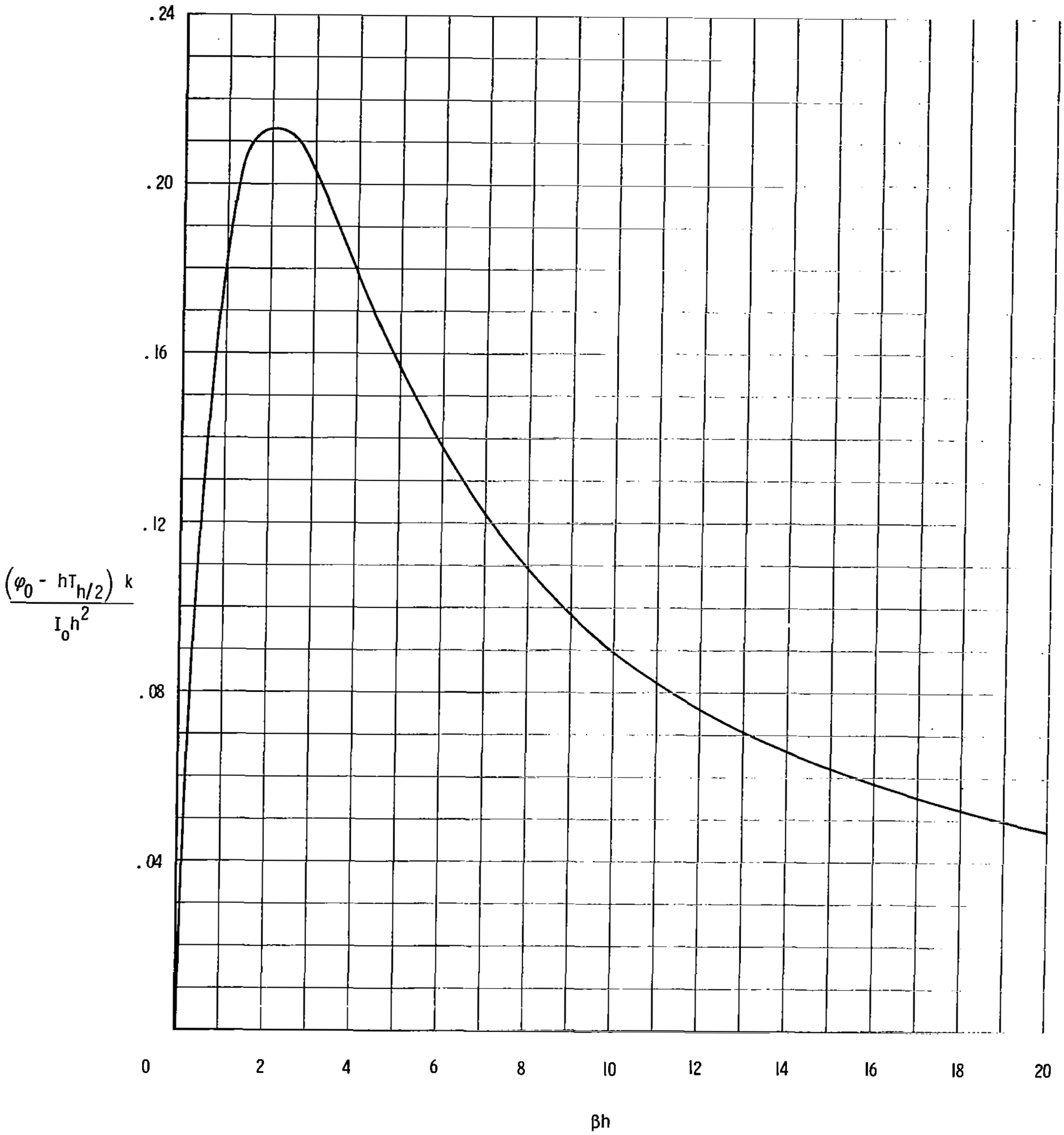
Figure 4.- Variation of maximum temperature differential through plate thickness as a function of βh. $\left(\frac{\partial T}{\partial z} = 0 \text{ at } z = -\frac{h}{2}\right)$

The plate temperature parameters φ_0 and φ_1 are obtained by appropriately integrating equation (40). The resulting equations are

$$\varphi_0 = hT_{h/2} + \frac{I_o h^2}{2k(\beta h)^2}\left\{e^{-\beta h}\left[2 - (\beta h)^2\right] + 2\beta h - 2\right\} \tag{41}$$

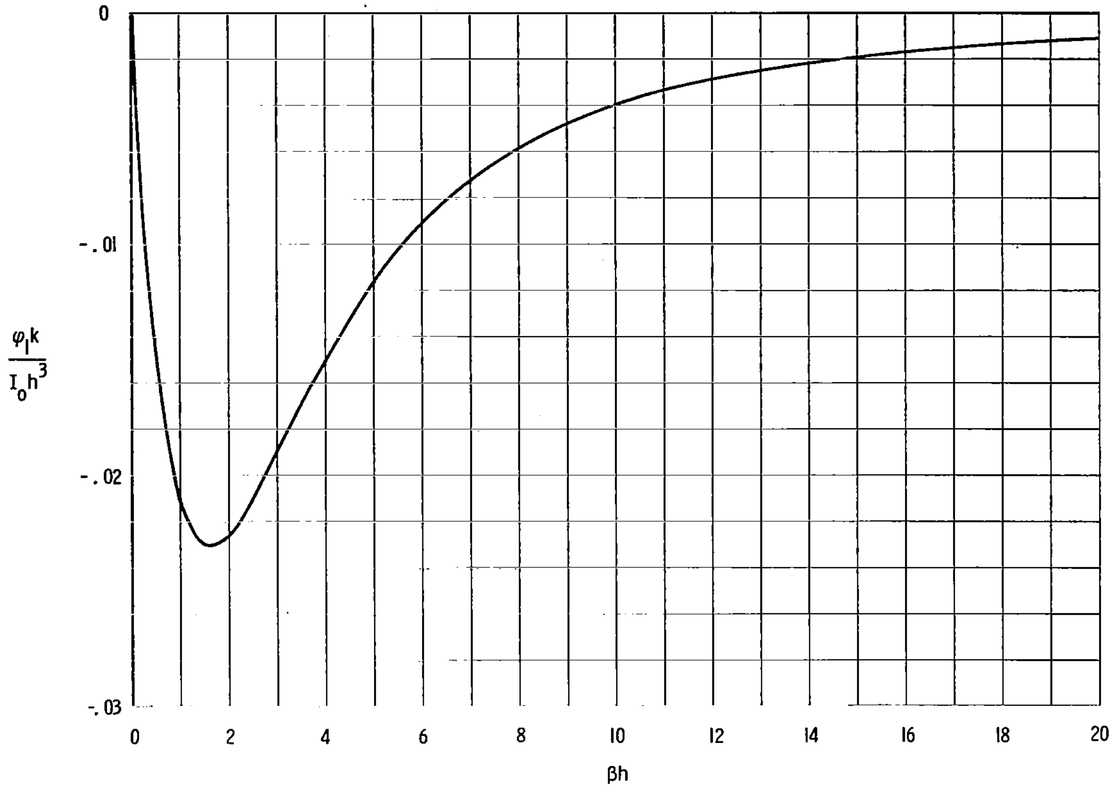
$$\varphi_1 = \frac{I_o h^3}{k(\beta h)^3}\left\{\frac{e^{-\beta h}}{12}\left[(\beta h)^3 - 6\beta h - 12\right] + 1 - \frac{\beta h}{2}\right\} \tag{42}$$

Figure 5(a) shows the variation of $\left(\varphi_0 - hT_{h/2}\right)k/I_o h^2$ with βh as obtained from equation (41), and indicates that the largest effect of radiation absorption on the value of φ_0 occurs for $\beta h = 2$. Figure 5(b) shows the variation of $\varphi_1 k/I_o h^3$ with βh as obtained from equation (42), and indicates that the maximum effect of radiation absorption on the value of φ_1 occurs when the value of βh is 1.61.



(a) Effect of βh on φ_0.

Figure 5.- Effect of βh on the plate temperature parameters when the temperature distribution and incident energy distribution are specified on the surface $z = h/2$, and when $\partial T/\partial z = 0$ at $z = -h/2$.



(b) Effect of βh on φ_1.

Figure 5.- Concluded.

CONCLUDING REMARKS

General expressions for the steady-state temperature distribution within plates have been obtained for conditions involving heat generation within the plate. The solutions presented satisfy the temperature boundary conditions on the large surfaces of the plate but do not usually satisfy the temperature boundary conditions at the edges of the plate.

The two temperature parameters that govern plate deformation are denoted by φ_0 and φ_1 and are respectively proportional to the average temperature through the plate thickness and the first moment of the temperature distribution in the thickness direction with respect to the midplane of the plate. The equations indicate that when the internal heating function is represented by an even function with respect to the midplane of the plate, the parameter φ_1 is unaffected by heat generation. The solution for constant heat generation is presented as an example of this case. If the internal heating function is represented by an odd function, φ_0 is unaffected by heat generation.

When the generation of heat is induced by the absorption of electromagnetic radiation, the resulting nonlinear temperature distribution through the plate thickness affects both the plate deformation parameters. The variation of the deformation parameters with respect to the dimensionless product βh (β is the linear absorption coefficient and h is the plate thickness) is of particular interest inasmuch as the magnitudes of β and h are dependent on the plate material and plate thickness, respectively. For the case in which the temperature boundary conditions and the incident radiation are harmonic functions, it is observed that βh has the greatest effect on the magnitudes of φ_0 and φ_1 for values of βh between zero and approximately 7. Therefore, if it is desirable to minimize the effect of internal absorption of electromagnetic radiation on thermal deformation, it is necessary to select a material and thickness for which the magnitude of the product βh is either very nearly zero or much greater than 7.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., August 28, 1967,
124-08-01-13-23.

APPENDIX A

DETERMINATION OF COEFFICIENTS b_0 AND b_1

The temperature distribution within the plate as defined by equation (5) is dependent on the unknown midplane temperature functions b_0 and b_1. Equations (10) and (11) relate b_0 and b_1 to the known boundary temperature functions and heat-generating function but are not directly solvable for b_0 and b_1 unless $\nabla^2 b_0 = \nabla^2 b_1 = 0$. Expressions for b_0 and b_1 are found in terms of the known functions $T_o{}^*$ and $P_o{}^*$, however, by appropriately reversing the series equations for $T_o{}^*$ and $P_o{}^*$. For example, a set of equations involving $T_o{}^*$ and its derivatives may be obtained in the following form:

$$
\left.
\begin{aligned}
T_o{}^* &= b_0 - \frac{\nabla^2 b_0}{2!}\left(\frac{h}{2}\right)^2 + \frac{\nabla^4 b_0}{4!}\left(\frac{h}{2}\right)^4 - \frac{\nabla^6 b_0}{6!}\left(\frac{h}{2}\right)^6 + \frac{\nabla^8 b_0}{8!}\left(\frac{h}{2}\right)^8 - \dots \\
\frac{C_2}{2!}\left(\frac{h}{2}\right)^2 \nabla^2 T_o{}^* &= C_2 \frac{\nabla^2 b_0}{2!}\left(\frac{h}{2}\right)^2 - \frac{C_2}{2!}\frac{\nabla^4 b_0}{2!}\left(\frac{h}{2}\right)^4 + \frac{C_2}{2!}\frac{\nabla^6 b_0}{4!}\left(\frac{h}{2}\right)^6 - \frac{C_2}{2!}\frac{\nabla^8 b_0}{6!}\left(\frac{h}{2}\right)^8 + \dots \\
\frac{C_4}{4!}\left(\frac{h}{2}\right)^4 \nabla^4 T_o{}^* &= C_4 \frac{\nabla^4 b_0}{4!}\left(\frac{h}{2}\right)^4 - \frac{C_4}{4!}\frac{\nabla^6 b_0}{2!}\left(\frac{h}{2}\right)^6 + \frac{C_4}{4!}\frac{\nabla^8 b_0}{4!}\left(\frac{h}{2}\right)^8 - \dots \\
\frac{C_6}{6!}\left(\frac{h}{2}\right)^6 \nabla^6 T_o{}^* &= C_6 \frac{\nabla^6 b_0}{2!}\left(\frac{h}{2}\right)^6 - \frac{C_6}{6!}\frac{\nabla^8 b_0}{2!}\left(\frac{h}{2}\right)^8 + \dots \\
&\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot
\end{aligned}
\right\} \tag{A1}
$$

where C_n are arbitrary constants. If these constants satisfy the set of equations given by

$$
\left.
\begin{aligned}
&C_2 - 1 = 0 \\
&C_4 - \frac{C_2(4)(3)}{2!} + 1 = 0 \\
&C_6 - \frac{C_4(6)(5)}{2!} + \frac{C_2(6)(5)}{2!} - 1 = 0 \\
&C_8 - \frac{C_6(8)(7)}{2!} + \frac{C_4(8)(7)(6)(5)}{4!} - \frac{C_2(8)(7)}{2!} + 1 = 0 \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot
\end{aligned}
\right\} \tag{A2}
$$

then the summation of equations (A1) is simply

$$b_0 = \sum_{n=0,2,4}^{\infty} \frac{C_n}{n!}\left(\frac{h}{2}\right)^n \nabla^n T_o^* \tag{A3}$$

Equations (A2) are of the form presented in reference 14, series (1145), and it is immediately apparent that equations (A2) are satisfied if

$$C_n = E_n \tag{A4}$$

where the constants E_n are Euler numbers and are defined in reference 14, pages 238 and 239. Thus, equation (A3) becomes

$$b_0 = \sum_{n=0,2,4}^{\infty} \frac{E_n}{n!}\left(\frac{h}{2}\right)^n \nabla^n T_o^* \tag{A5}$$

In determining b_1 in terms of P_o, a set of equations may be written in the form

$$\left.\begin{aligned}
P_o^* &= \frac{b_1 h}{2} - \frac{\nabla^2 b_1}{3!}\left(\frac{h}{2}\right)^3 + \frac{\nabla^4 b_1}{5!}\left(\frac{h}{2}\right)^5 - \frac{\nabla^6 b_1}{7!}\left(\frac{h}{2}\right)^7 + \frac{\nabla^8 b_1}{9!}\left(\frac{h}{2}\right)^9 - \dots \\
\frac{D_1}{2!}\left(\frac{h}{2}\right)^2 \nabla^2 P_o^* &= D_1 \frac{\nabla^2 b_1}{2!}\left(\frac{h}{2}\right)^3 - D_1 \frac{\nabla^4 b_1}{2!3!}\left(\frac{h}{2}\right)^5 + D_1 \frac{\nabla^6 b_1}{2!5!}\left(\frac{h}{2}\right)^7 - D_1 \frac{\nabla^8 b_1}{2!7!}\left(\frac{h}{2}\right)^9 + \dots \\
\frac{D_2}{4!}\left(\frac{h}{2}\right)^4 \nabla^4 P_o^* &= D_2 \frac{\nabla^4 b_1}{4!}\left(\frac{h}{2}\right)^5 - D_2 \frac{\nabla^6 b_1}{4!3!}\left(\frac{h}{2}\right)^7 + D_2 \frac{\nabla^8 b_1}{4!5!}\left(\frac{h}{2}\right)^9 - \dots \\
\frac{D_3}{6!}\left(\frac{h}{2}\right)^6 \nabla^6 P_o^* &= D_3 \frac{\nabla^6 b_1}{6!}\left(\frac{h}{2}\right)^7 - D_3 \frac{\nabla^8 b_1}{6!3!}\left(\frac{h}{2}\right)^9 + \dots \\
&\cdots
\end{aligned}\right\} \tag{A6}$$

where the constants D_m are arbitrary constants. If these constants satisfy the set of equations given by

$$\left.\begin{aligned}
&\frac{D_1}{2!} - \frac{1}{3!} = 0 \\
&\frac{D_2}{4!} - \frac{D_1}{2!3!} + \frac{1}{5!} = 0 \\
&\frac{D_3}{6!} - \frac{D_2}{4!3!} + \frac{D_1}{2!5!} - \frac{1}{7!} = 0 \\
&\frac{D_4}{8!} - \frac{D_3}{6!3!} + \frac{D_2}{4!5!} - \frac{D_1}{2!7!} + \frac{1}{9!} = 0 \\
&\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot
\end{aligned}\right\} \tag{A7}$$

then the summation of equations (A1) is simply

$$\frac{b_1 h}{2} = P_o^* + \sum_{n=3,5,7}^{\infty} \frac{D_{(n-1)/2}}{(n-1)!}\left(\frac{h}{2}\right)^{n-1} \nabla^{(n-1)} P_o^* \tag{A8}$$

By using series (1135) of reference 14 along with the identities

$$\left.\begin{aligned}
&\beta_n = \left(2^{2n} - 2\right)B_n && \text{for all } n \\
&B_n(1) = 0 && \text{for all } n \\
&B_n\left(\frac{1}{2}\right) = 0 && \text{for odd } n
\end{aligned}\right\} \tag{A9}$$

which are listed respectively in series (1130), (1136), and (1142) of reference 14, it may be shown that, for odd values of $n \geqq 3$

$$0 = -\frac{1}{n!} + \frac{\beta_1}{2!(n-2)!} - \frac{\beta_2}{4!(n-4)!} + \ldots - \frac{(-1)^{(n-1)/2}\beta_{(n-1)/2}}{(n-1)!} \tag{A10}$$

Equations obtained from (A10) have the same form as equations (A7). Equations (A7) are therefore satisfied if

$$D_{(n-1)/2} = \beta_{(n-1)/2} \tag{A11}$$

where the constants $\beta_{(n-1)/2}$ are related to the Bernoulli numbers as indicated by equation (A9). Thus equation (A8) becomes

$$b_1 = \frac{2P_o^*}{h} + \sum_{n=3,5,7}^{\infty} \frac{\beta_{(n-1)/2}}{(n-1)!}\left(\frac{h}{2}\right)^{n-2} \nabla^{n-1} P_o^* \tag{A12}$$

APPENDIX B

SIMPLIFICATION OF TEMPERATURE EQUATIONS

Temperature Distributions in Which $T_{h/2}$, $T_{-h/2}$, and a_n Are Biharmonic Functions

When completely arbitrary temperature boundary conditions and heating function are introduced into the general series solution for the temperature distribution (eq. (5)) and into the series for b_0 and b_1 (eqs. (12) and (13)), the mechanics of solution become unwieldy. However, if the boundary temperatures $T_{h/2}$, $T_{-h/2}$, and the heating-function coefficients $a_n(x,y)$ are restricted so that

$$\left.\begin{aligned} \nabla^m T_{h/2} &= 0 \\ \nabla^m T_{-h/2} &= 0 \\ \nabla^m a_n &= 0 \end{aligned}\right\} \tag{B1}$$

where m is some reasonably small, even integer, then the solution for the temperature distribution is more easily obtainable. For example, if it is assumed that $T_{h/2}$, $T_{-h/2}$, and a_n are biharmonic functions then,

$$\left.\begin{aligned} \nabla^4 T_{h/2} &= 0 \\ \nabla^4 T_{-h/2} &= 0 \end{aligned}\right\} \tag{B2}$$

$$\nabla^4 a_n = 0 \tag{B3}$$

Applying equations (B2) and (B3) to equations (6) to (9), (12), and (13) gives:

$$\nabla^4 T_o = 0 \tag{B4}$$

$$\nabla^4 P_o = 0 \tag{B5}$$

$$\nabla^4 T_o{}^* = 0 \tag{B6}$$

$$\nabla^4 P_o{}^* = 0 \tag{B7}$$

$$\nabla^4 b_0 = 0 \tag{B8}$$

$$\nabla^4 b_1 = 0 \tag{B9}$$

Substituting equations (B3), (B8), and (B9) into the general equation for the temperature distribution (eq. (5)) gives

$$T = b_0 - \frac{\nabla^2 b_0}{2} z^2 + b_1 z - \frac{\nabla^2 b_1}{6} z^3 + \frac{a_0}{2} z^2 + \frac{a_1}{6} z^3 + \sum_{n=2,3,4}^{\infty} z^{n+2}\left[\frac{a_n}{(n+2)(n+1)} - \frac{\nabla^2 a_{(n-2)}}{(n+2)(n+1)(n)(n-1)}\right] \tag{B10}$$

where b_0 and b_1 are obtained from equations (12) and (13) as

$$b_0 = T_o^* + \frac{\nabla^2 T_o^*}{2}\left(\frac{h}{2}\right)^2 \tag{B11}$$

$$b_1 = \frac{2P_o^*}{h} + \frac{h}{12}\nabla^2 P_o^* \tag{B12}$$

Expressions for T_o^* and P_o^* are obtained from equations (8) and (9) in terms of T_o, P_o, and a_n and are given by

$$T_o^* = T_o - \frac{a_0 h^2}{8} - \sum_{n=2,4,6}^{\infty}\left(\frac{h}{2}\right)^{n+2}\left[\frac{a_n}{(n+2)(n+1)} - \frac{\nabla^2 a_{(n-2)}}{(n+2)(n+1)(n)(n-1)}\right] \tag{B13}$$

$$P_o^* = P_o - \frac{a_1 h^3}{48} - \sum_{n=3,5,7}^{\infty}\left(\frac{h}{2}\right)^{n+2}\left[\frac{a_n}{(n+2)(n+1)} - \frac{\nabla^2 a_{(n-2)}}{(n+2)(n+1)(n)(n-1)}\right] \tag{B14}$$

The temperature distribution may now be written in terms of T_o, P_o, and a_n by eliminating T_o^* and P_o^* from the expressions for b_0 and b_1 (eqs. (B11) and (B12)) and substituting the resulting expressions for b_0 and b_1 into equation (B10). This procedure gives the temperature distribution as

$$\begin{aligned}
T = T_o &- \sum_{n=0,2,4}^{\infty}\left(\frac{h}{2}\right)^{n+2}\frac{a_n}{(n+2)(n+1)} + \sum_{n=0,2,4}^{\infty}\left(\frac{h}{2}\right)^{n+4}\frac{\nabla^2 a_n}{(n+4)(n+3)(n+2)(n+1)} \\
&+ \left[\left(\frac{h}{2}\right)^2 - z^2\right]\left[\frac{\nabla^2 T_o}{2} - \frac{1}{2}\sum_{n=0,2,4}^{\infty}\left(\frac{h}{2}\right)^{n+2}\frac{\nabla^2 a_n}{(n+2)(n+1)} + \frac{z}{3h}\nabla^2 P_o \right. \\
&\left. - \frac{z}{3h}\sum_{n=1,3,5}^{\infty}\left(\frac{h}{2}\right)^{n+2}\frac{\nabla^2 a_n}{(n+2)(n+1)}\right] + \sum_{n=0,1,2}^{\infty} z^{n+2}\frac{a_n}{(n+2)(n+1)} \\
&- \sum_{n=0,1,2}^{\infty} z^{n+4}\frac{\nabla^2 a_n}{(n+4)(n+3)(n+2)(n+1)} + \frac{2z}{h}\left[P_o - \sum_{n=1,3,5}^{\infty}\left(\frac{h}{2}\right)^{n+2}\frac{a_n}{(n+2)(n+1)}\right. \\
&\left. + \sum_{n=1,3,5}^{\infty}\left(\frac{h}{2}\right)^{n+4}\frac{\nabla^2 a_n}{(n+4)(n+3)(n+2)(n+1)}\right]
\end{aligned} \tag{B15}$$

APPENDIX B

Expressions for φ_0 and φ_1 may be obtained by appropriately integrating equation (B15) with respect to z (see eqs. (14) and (15)).

Solutions in Which $T_{h/2}$, $T_{-h/2}$, and a_n Are Harmonic Functions

If the further restriction is made that $T_{h/2}$, $T_{-h/2}$, and a_n are harmonic functions, then the ∇^2 terms appearing in equations (B10) to (B15) vanish and the temperature distribution becomes

$$T = T_o + \frac{2P_oz}{h} + \sum_{n=0,2,4}^{\infty} \frac{a_n\left[z^{n+2} - \left(\frac{h}{2}\right)^{n+2}\right]}{(n+1)(n+2)} + z\sum_{n=1,3,5}^{\infty} \frac{a_n\left[z^{n+1} - \left(\frac{h}{2}\right)^{n+1}\right]}{(n+1)(n+2)} \tag{B16}$$

Applying these results to the plate thermal deformation parameters φ_0 and φ_1 (eqs. (14) and (15)) gives

$$\varphi_0 = T_oh - 2\sum_{n=0,2,4}^{\infty} \frac{\left(\frac{h}{2}\right)^{n+3} a_n}{(n+1)(n+3)} \tag{B17}$$

$$\varphi_1 = \frac{P_oh^2}{6} - \frac{2}{3}\sum_{n=1,3,5}^{\infty} \frac{\left(\frac{h}{2}\right)^{n+4} a_n}{(n+2)(n+4)} \tag{B18}$$

Biharmonic Solution for the Case of Absorption of Electromagnetic Radiation

The temperature distributions caused by the absorption of electromagnetic radiation are obtained by substituting the expression for a_n, given by equation (29), into equation (B15). This gives

$$T = T_o + \frac{2P_oz}{h} + \left[\left(\frac{h}{2}\right)^2 - z^2\right]\left\{\frac{\nabla^2 T_o}{2} + \frac{\nabla^2 P_oz}{3h} + \frac{\nabla^2 I_o}{k\beta}\left[\frac{1}{4}\left(1 + e^{-\beta h}\right) + \frac{z}{6h}\left(1 - e^{-\beta h}\right)\right]\right\}$$
$$+ \left(\frac{I_o}{k\beta} - \frac{\nabla^2 I_o}{k\beta^3}\right)\left[\frac{1}{h}\left(\frac{h}{2} + z\right) + \frac{e^{-\beta h}}{h}\left(\frac{h}{2} - z\right) - e^{-\beta\left(\frac{h}{2} - z\right)}\right] \tag{B19}$$

where it should be noted that $\nabla^4 I_o$ must be zero because of the requirement that $\nabla^4 a_n$ be zero (see eq. (29)). Appropriately integrating equation (B19) with respect to z gives φ_0 and φ_1 as

$$\varphi_0 = T_o h + \left(\frac{h}{2}\right)^3 \left[\frac{2}{3}\nabla^2 T_o + \frac{\nabla^2 I_o}{3k\beta}\left(1 + e^{-\beta h}\right)\right]$$

$$+ \left(\frac{I_o}{k\beta} - \frac{\nabla^2 I_o}{k\beta^3}\right)\left[\frac{h}{2}\left(1 + e^{-\beta h}\right) - \frac{1 - e^{-\beta h}}{\beta}\right] \tag{B20}$$

$$\varphi_1 = \frac{P_o h^2}{6} + \frac{h^2}{720}\left[2h^2\nabla^2 P_o + \frac{h^2\nabla^2 I_o}{k\beta}\left(1 - e^{-\beta h}\right)\right]$$

$$+ \left(\frac{I_o}{k\beta} - \frac{\nabla^2 I_o}{k\beta^3}\right)\left\{\frac{h^2}{12}\left(1 - e^{-\beta h}\right) - \frac{1}{\beta^2}\left[\frac{\beta h}{2} - 1 + e^{-\beta h}\left(\frac{\beta h}{2} + 1\right)\right]\right\} \tag{B21}$$

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